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Renovated Ballistic Equation of Ejected Blocks and Its Application to the 1982 and 1983 Sakurajima Eruptions

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We propose a ballistic equation of ejected volcanic blocks that is renovated by considering air resistance, wind velocity and direction dependence of the initial velocity. In the equation, we treat the air resistance as a vector that coincides with the opposite direction of flight. By comparing the calculated results based on the equation of motion and the observed data, such as spatial distribution of landed blocks, initial velocities by successive photos and so on, we tried to reproduce kinematic aspects of the ejected blocks at the 1982 and 1983 eruptions of Sakurajima Volcano, Kyushu, Japan.

Key words: ballistic equation of volcanic blocks, air resistance as a vector, Sakurajima volcano

1. Introduction

Volcano-energetic study is very important for discussing the eruption process. In addition to the energy release due to the crustal deformation, seismic, and plume rise activities, estimate of the volcanic block discharge energy is important not only for volcanological standpoints but also for preventing disasters. For these purposes, we need to evaluate the velocity, direction and mass distribution of the ejected blocks at the exit of a crater. However, in most cases, it is hard to measure directly such quantities during an eruption. Therefore, the distribution of volcanic blocks landed on and around the crater has been investigated after the eruptions to reproduce the kinetic aspects of the ejected blocks.

Ballistic curves of volcanic blocks are controlled by effects such as the direction of the principal axis of ejection, the velocity and direction of the wind, the air resistance to the blocks and the direction dependence of the initial velocity of ejection. Among these effects, the air resistance plays an important role for accurately estimating the initial velocity. Matuzawa (1934) pointed out that the air resistance applied to a flying object should be treated as a vector. However, previous works (*e.g.*, Nagata, 1938; Minakami, 1942; Iguchi *et al*., 1983; Iguchi and Kamo, 1984) did not take account of this problem.

In the present paper, we numerically calculate synthetic ballistic curves of volcanic blocks for various combinations of the above-mentioned parameters including the air resistance treated as a vector. Referring to these results, we tried to reproduce detailed kinematic aspects of the ejected blocks at the 1982 and 1983 eruptions of Sakurajima Volcano, Kyushu, Japan.

2. Ballistic equation of ejected blocks

In this chapter, we count up some important factors which affect the ballistic curves of ejected blocks: air resistance, wind velocity, direction of the initial velocity of ejection, inclination of the principal axis of explosion and size of the blocks.

2-1 Equation of motion considering the air resistance

Matuzawa (1934: in a revised paper of Matuzawa, 1933) pointed out that the air resistance working to a flying object should be treated as a vector, although he did not present any detailed form. Then, we formulate an equation of the motion of a flying object following the Matuzawa's suggestion. In the present paper, we treat intensity of the air resistance is in proportion to the second power of the block velocity, V^2, while the direction of the applied force is opposite to that of the flight, $-\boldsymbol{V}$. Thus, we propose the equation of motion as:

$$m\frac{d\boldsymbol{V}}{dt}=m\boldsymbol{g}+\boldsymbol{f}, \quad (1)$$

where m, $\boldsymbol{V}$, and $\boldsymbol{g}$ are respectively the mass of block, the flying velocity of block and the acceleration of gravity, respectively. $\boldsymbol{f}$ means air resistance vector. The inertial air resistance $\boldsymbol{f}$ is expressed as:

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$$\boldsymbol{f} = -\kappa' V^2 \left(\frac{\boldsymbol{V}}{V}\right) = -\kappa' V \cdot \boldsymbol{V}, \tag{2}$$

where κ' is the coefficient of the air resistance, that is, $f = \kappa' V^2$. When we select the Cartesian co-ordinate so that the earth's surface is the plane $z=0$ and Z-axis is positive in the vertically upward direction, the following relation holds as:

$$\boldsymbol{V} = V_x \boldsymbol{i} + V_y \boldsymbol{j} + V_z \boldsymbol{k} \ \ and \ \ \boldsymbol{g} = -g\boldsymbol{k}, \tag{3}$$

where i, j, k are the unit vectors along X, Y, Z axes, respectively. Taking $\kappa'/m = \kappa$, the equation of motion is then governed by:

$$\frac{d\boldsymbol{V}}{dt} = -g\boldsymbol{k} - \kappa V \cdot \boldsymbol{V}. \tag{4}$$

The detailed expression of the air resistance f was proposed by Wilson and Huang (1979) as:

$$f = \kappa' V^2 = \frac{1}{2} C_D \rho_A S V^2, \tag{5}$$

where S, V and ρ_A denote respectively the cross-sectional area, velocity of a volcanic block and the air density. As $\kappa' = m\kappa = \frac{1}{2} C_D \rho_A S$, κ, becomes $\frac{3}{4}\frac{C_D \rho_A}{\rho' d}$ assuming a spherical block with a diameter d and a density ρ'. The altitude dependence of the air density ρ_A is given as a function of z as follows (Matuzawa, 1933):

$$\rho_A = \rho_0 \left(1 - \frac{\beta}{T_0} z\right)^A, \tag{6}$$

where ρ_0 and T_0 are the air density and the air temperature of the standard atmosphere at $z=0$ when the crater is assumed to exist at the sea level, respectively. β denotes the vertical temperature gradient of the air. A is defined as $\frac{\rho_0 T_0}{P_0 \beta} g$ when P_0 is the atmospheric pressure at the sea level. Therefore, coefficient of the air resistance κ at altitude z is expressed as:

$$\kappa = \lambda \left(1 - \frac{\beta}{T_0} z\right)^A, \tag{7}$$

when λ is given as:

$$\lambda = \frac{3}{4} \frac{C_D \rho_0}{\rho' d}. \tag{8}$$

Wilson and Huang (1979) obtained an empirical relation to express C_D in eq. (5) through laboratory experiments of falling volcanic fragments as follows:

$$C_D = (24/R_e) F^{-0.828} + 2\sqrt{1.07 - F}, \tag{9}$$

where R_e means the Reynolds number $(= Vd\,\rho/\eta)$. η and ρ denote respectively the coefficients of kinematic viscosity and density of the air. F represents shape parameter and $F = (b+c)/2a$ when the object is assumed to be an ellipsoid of which diameters are a, b and c along three axes. Although R_e is less than 10^3 when volcanic block of 1 cm in diameter flies in the air with velocity below 1 m/s, it is reasonable to assume that the second term is dominant in comparison to the first term for relatively large volcanic blocks:

$$C_D = 2\sqrt{1.07 - F}. \tag{10}$$

Substituting C_D into eq. (8),

$$\lambda = \frac{3}{2}\sqrt{1.07 - F}\left(\frac{\rho_0}{\rho' d}\right). \tag{11}$$

Coefficient $\frac{3}{2}$ in the above equation is given by assuming that block shapes are spherical. Hereafter, expanding to realistic cases, we express this coefficient in terms of K_D to be determined by analysis of field data or wind tunnel experiments:

$$\lambda = K_D \sqrt{1.07 - F}\left(\frac{\rho_0}{\rho' d}\right). \tag{12}$$

Therefore, coefficient of the air resistance κ in eq. (7) becomes:

$$\kappa = \frac{K_D \rho_0 \sqrt{1.07 - F}}{\rho' d} \left\{1 - \frac{\beta}{T_0}(H + z)\right\}^A. \tag{13}$$

Here, H and z denote respectively the altitude of the bottom of the crater and the height of a flying block measured from the bottom of the crater. This equation indicates that the air resistance is inversely proportional to a diameter of block d and proportional to the coefficient K_D. In eq. (13), the values with suffix 0 represent those of the standard atmosphere at the altitude of H.

2-2 Effect of wind velocity on the motion of volcanic blocks

We consider the effect of wind velocity on the motion of volcanic blocks by referring to Minakami (1942). Let $\boldsymbol{U}$ and $\boldsymbol{V}$ be respectively the wind velocity vector and the block velocity vector. Then, the relative velocity vector $\boldsymbol{W}$ is given by:

$$\boldsymbol{W} = \boldsymbol{V} - \boldsymbol{U}. \tag{14}$$

As we assume that the air resistance is proportional to the second power of the relative velocity $\boldsymbol{W}$, the air resistance vector $\boldsymbol{f}$ is expressed as:

$$\boldsymbol{f} = -\kappa' W^2 \left(\frac{\boldsymbol{W}}{W}\right) = -\kappa' W \cdot \boldsymbol{W}. \tag{15}$$

Putting $\boldsymbol{V} = V_x \boldsymbol{i} + V_y \boldsymbol{j} + V_z \boldsymbol{k}$ and $\boldsymbol{U} = U_x \boldsymbol{i} + U_y \boldsymbol{j}$, and assuming a horizontal wind field, the relative velocity vector $\boldsymbol{W}$ is represented as:

$$\boldsymbol{W} = (V_x - U_x)\boldsymbol{i} + (V_y - U_y)\boldsymbol{j} + V_z \boldsymbol{k}. \tag{16}$$

Therefore, we have an equation of motion under the wind velocity $\boldsymbol{U}$ as:

$$m\left(\frac{d\boldsymbol{V}}{dt}\right) = m\boldsymbol{g} - \kappa' W \cdot \boldsymbol{W}, \tag{17}$$

where $W = \sqrt{(V_x - U_x)^2 + (V_y - U_y)^2 + V_z^2}$. m and $\boldsymbol{g}$

denote respectively the mass of block and the acceleration of gravity. Taking $\kappa'/m=\kappa$, as in Section 2–1, eq. (17) is expressed as:

$$\frac{d\boldsymbol{V}}{dt}=-g\boldsymbol{k}-\kappa W\cdot\boldsymbol{W}. \tag{18}$$

By specifically expressing eq. (18) with Cartesian coordinate, we can write the equation of motion by the following simultaneous differential equations:

$$\left.\begin{aligned}&\frac{d^2x}{dt^2}+\kappa W\left(\frac{dx}{dt}-U_x\right)=0\\&\frac{d^2y}{dt^2}+\kappa W\left(\frac{dy}{dt}-U_y\right)=0\\&\frac{d^2z}{dt^2}+\kappa W\left(\frac{dz}{dt}\right)+g=0\end{aligned}\right\} \tag{19}$$

where $W=\sqrt{(dx/dt-U_x)^2+(dy/dt-U_y)^2+(dz/dt)^2}$. We could calculate the ballistic curve of the ejected block by integrating these equations numerically by giving initial condition V_0.

2–3 Direction dependence on the initial velocity of ejected blocks

Iguchi *et al*. (1983) took successive photos during nighttime to trace trajectories of volcanic blocks ejected by 5 eruptions of Sakurajima Volcano in 1982–1983. The photo analyses revealed that vertically ejected blocks had the maximum initial velocity (V_{max}). Then, referring to a study by Штейнбергб (1975) on shock waves arising from explosions, they examined the relation between the ejection angle (e) of the blocks and the initial velocity (V) normalized to V_{max} and found that the maximum initial velocity for each ejection angle holds the following empirical relation as: sin

$$V=V_{max}\sin^{1.5}e, \tag{20}$$

where 'e' is the angle taken from the horizontal plane (Fig. 8 in Iguchi *et al.*, 1983). This equation is presented by assuming that the explosion principal axis (Fig. 1) is vertical. In the photo analyses, all the data from the 5 eruptions are collectively used. However, when we re-examine the relation V/V_{max} versus ejection angle for each eruption, the index number in the above equation seems to take not only a value of 1.5 but also other values as shown in Fig. 2:

Jun. 8, 1982 eruption: 2.5,
Jul. 23, 1982 eruption: 3.0,
Aug. 7, 1982 eruption: 1.5,
Nov. 1, 1982 eruption: 1.5,
Jan. 10, 1983 eruption: 6.0.

Then, taking into consideration in cases of the inclined explosion principal axis, we revise the definition of the direction dependence of the initial velocity of ejected blocks (V_0) using a variable index M instead of a constant value of 1.5 in eq. (20) as:

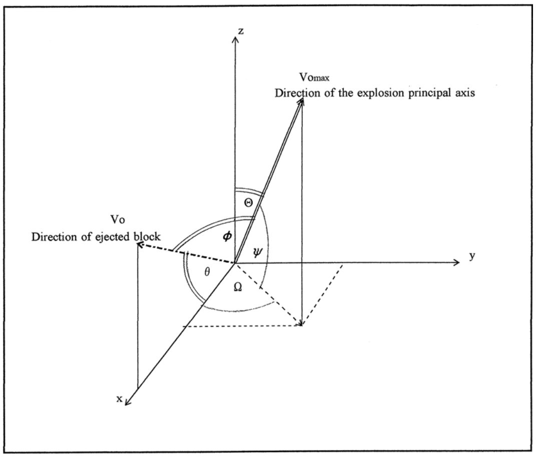


Fig. 1. Diagram of the angles showing the direction of explosion principal axis and the direction of an ejected block. The direction of the component on the horizontal plane of the initial velocity V_0 of an ejected block is chosen as x-axis. z-axis is zenith direction and x, y and z axes chose Cartesian coordinate system. The angles in this figure are as follows:
The azimuth of the explosion principal axis is measured clockwise from the north.
Θ: The angle between the explosion principal axis and z-axis.
Ω: The difference between the azimuth of the explosion principal axis and that of an ejected block.
ϕ: The angle between the explosion principal axis and the ejected direction of a block.
ψ ($=90°-\Theta$): The angle between the explosion principal axis and the horizontal plane.
θ: The angle between the direction of the initial velocity V_0 of a block and x-axis.

$$V_0(\phi)=V_{0max}\cos^M\phi, \tag{21}$$

where $V_{0\,max}$ is the maximum initial velocity of the block (Suzuki *et al.*, 2008a). We call M the '*angle dependent parameter'*. This parameter is not for volcanological meaning but only for empirical one. When the initial elevation angle of an ejected block is θ, the angle between the directions of the explosion principal axis and the ejected block, ϕ, is expressed as:

$$\cos\phi=\cos\psi\cos\Omega\cos\theta+\sin\psi\sin\theta. \tag{22}$$

A representation of the angles (ϕ, ψ, Ω, θ) is shown in Fig. 1. One should note that the x-axis is taken to coincide with the horizontal component of the direction of an ejected block. Hereafter, for example, a case that the explosion principal axis has a inclination of 10° from vertical axis and has a horizontal component to N45° E is expressed as N45Θ10.

2-4 Diameter dependence on the initial velocity of ejected blocks

When we assume that the volcanic blocks of various sizes are thrown from a crater with the same initial velocity, the larger size blocks must fly to longer distances from the crater than the smaller blocks, because the air resistance varies inversely as the size of blocks as indicated by eq. (13) (Case 1). Therefore, in this case, the larger size blocks are expected to land dominantly in peripheral area of the spatially dispersed blocks. On the other hand, large size blocks were mixed frequently within smaller size group of landed blocks as observed in the 1938 eruption of Asama-yama Volcano (Minakami, 1942) and the 1977 eruption of Usu Volcano (Katsui *et al*., 1978) as shown in Fig. 3(a) and Fig. 3(b). These evidences may be an indication that the initial velocity depends on the block size (Case 2).

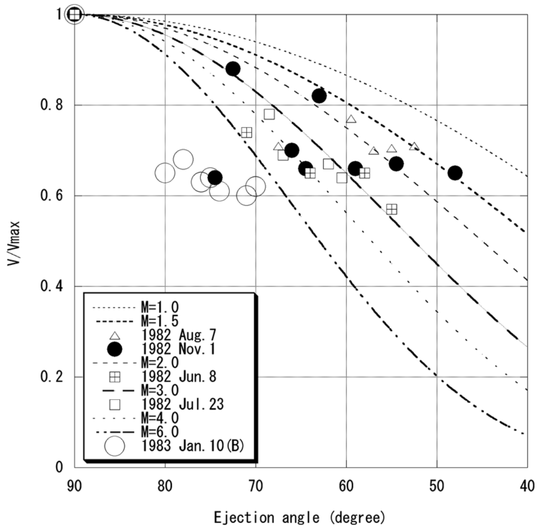


Fig. 2. Normalized initial velocity versus ejection angle based on the observed data of 5 eruptions in Sakurajima Volcano (Fig. 7 in Iguchi *et al*., 1983). The observed data are normalized to the maximum initial velocity of the each eruption. Using different symbols, the normalized data are shown in this figure respectively. Curves are calculated relations depending on parameter *M*.

We made numerical model calculations using the ballistic equation (eq. (9)) in order to interpret the size distribution of landed blocks as described above. Landing distance of the volcanic blocks depends on the initial velocity, discharge angle and size of block. Fig. 4 shows relations between the maximum landing distance and the initial velocity for various sizes of blocks. The landing distance is proportional to the initial velocity and the diameter of blocks, as a matter of course. Note that the air resistance K_D can be almost ignored for the blocks with landing distance less than several hundred meters.

Case 1: Assuming the constant initial velocity of 250 m/sec, we calculate the landing distance for various discharge angles to estimate the diameter dependence on the maximum landing distance. Calculated results reveal that the maximum landing distance increases monotonously with diameter of the blocks as shown in Fig. 5(a).

Case 2: Assuming that the smaller the size of block, the

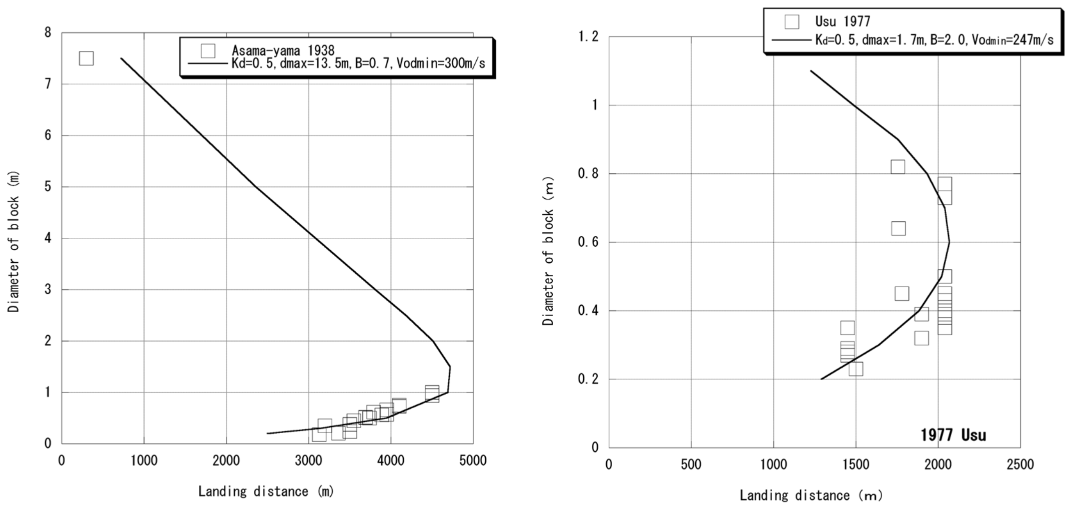


Fig. 3. (a) Open squares show the relation between the observed diameter of blocks and landing distances in case of the 1938 eruption of Asama-yama Volcano (Minakami, 1942), while the solid curve shows the calculated one. (b) Same as (a) for the 1977 eruption of Usu Volcano (Katsui *et al*., 1978). Solid curves in (a) and (b) are calculated respectively for $d_{max}=13.5$ m and $d_{max}=1.7$ m.

larger would be the initial velocity, we propose hypothetically a diameter dependent equation of the initial velocity V_0 as shown in eq. (23):

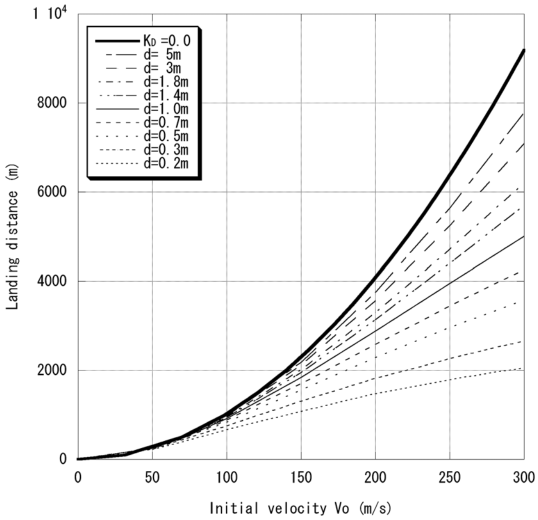


Fig. 4. Calculated relation between the initial velocity and the landing distance. Ten cases are shown for no air resistance $K_D=0$ (thick solid curve) and $K_D=0.5$ (other curves) depending on the diameter of blocks. Note that the air resistance K_D can be ignored for the blocks landed at distances less than several hundred meter.

$$V_{0max}(d)=V_{0dmin}\left\{1-\left(\frac{d}{d_{max}}\right)^B\right\}. \tag{23}$$

Let V_{0dmin} and B be respectively the initial velocity of a minute block with diameter d_{min} and the '*diameter dependent parameter*'. d_{max} represents the maximum size of block that can acquire the initial velocity; in other words, blocks with larger diameter than d_{max} cannot be thrown out from the crater. Fig. 5 (b) represents the calculated maximum landing distance in every block size. Blocks with middle size such as around 2 m in diameter land at the longest distance; blocks with larger diameter than about 2 m are mixed within the smaller size group. This tendency may be a proof of diameter dependence of the initial velocity, although assumed values of B, d_{max} and V_{0dmin} are only examples.

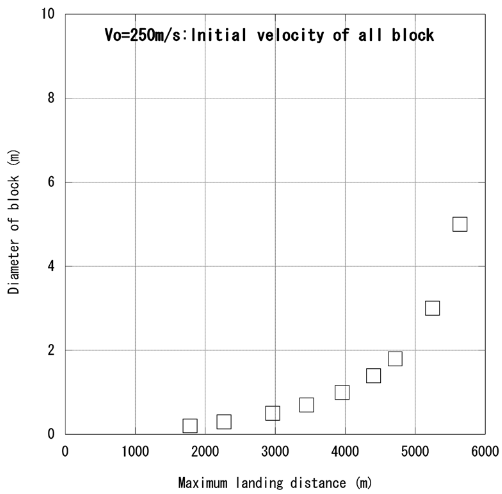
2-5 Combination of direction and diameter dependence of the initial velocity

Combining the direction and the diameter dependence of the initial velocity in eqs. (21) and (23), we obtain the following relation:

$$V_0(d,\phi)=V_{0dmin}\left\{1-\left(\frac{d}{d_{max}}\right)^B\right\}\cos^M\phi \tag{24}$$

3. Model calculations and their applications to eruptions of Sakurajima Volcano

We make model calculations of the ballistic curve of an ejected block, based on the equation of motion shown in eq. (19) that includes the effects of the air resistance, the wind velocity, the direction and diameter dependence of



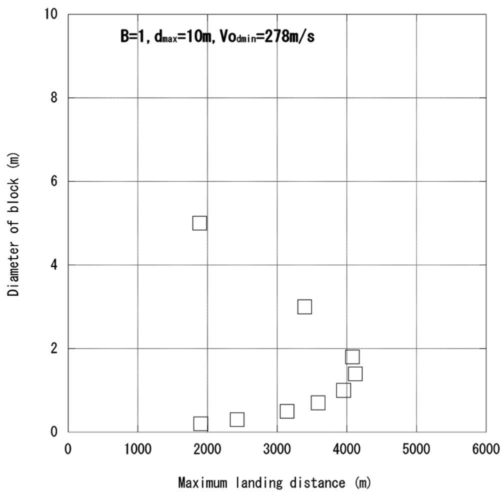


Fig. 5. (a) Calculated relation between diameter of volcanic blocks and the maximum landing distance, assuming the constant initial velocity of 250 m/sec. The maximum landing distance increases monotonously with diameter of the blocks. (b) Calculated relation between diameter of volcanic blocks and the maximum landing distance, assuming the diameter dependent initial velocity. Blocks with the middle size such as around 2 m in diameter land at the longest distance.

the initial velocity. In addition, the ballistic equation includes many parameters such as K_D , F, M, B, d_{max} etc. However, it may be very difficult to give constraint of each parameter because every eruption is probably a combination of different values of the various parameters. In addition, the observed field data are not necessarily sufficient to estimate the whole parameters: for examples, only the distribution of the landed blocks was observed in some cases while only initial conditions of direction and velocity of ejected blocks were observed in other cases.

We try to reproduce the detailed kinematic aspects of the ejected blocks at the 1982 and 1983 Sakurajima eruptions in order to interpret the observed field data. In calculations, the following constant values of the standard atmosphere are assumed as:

air density at the sea level: $\rho_0 = 1.226\,\mathrm{kg/m^3}$,
air temperature at the sea level: $T_0 = 288.15$ K,
air pressure at the sea level: $P_0 = 101325$ Pa,
vertical temperature gradient: $\beta = 0.0065$ K/m.

We take no account of local and seasonal changes of ρ_0, T_0 and P_0 because the changes observed in the Sakurajima region by the Kagoshima Local Meteorological Observatory (KLMO) show an insignificant effect less than about 2 % on the air resistance in comparison with that of the standard atmosphere. Mean aspect ratio of blocks F is assumed as 0.7 which is generally thought value (*e.g.*, Katsui *et al*., 1978). Prior to make model calculations for Sakurajima eruptions, we refer to some values of the parameters estimated from the analyses of the 1938 eruption of Asama-yama Volcano, the Central Japan, and the 1977 eruption of Usu Volcano, Hokkaido, Japan (Suzuki *et al*., 2008b) to reduce the degrees of freedom of the other parameters. The coefficient concerning the air resistance, K_D, is estimated as about 0.5 in both of the eruptions. This value is thought to be valid for other volcanic eruptions because the air resistance is the same everywhere. Therefore, we apply this value to the following three case studies on the Sakurajima eruptions. The mean density of the blocks, ρ', of Sakurajima Volcano was given by Iguchi *et al*. (1983) as 2.2×10^3 kg/m^3.

Unfortunately, the effect of the diameter dependence to the initial velocity is not included in the following model calculations because the size distribution of landed blocks was not measured after the Sakurajima eruptions (Iguchi *et al*., 1983; Iguchi and Kamo, 1984). Therefore, we apply eq. (21) for the initial velocity.

3-1 The Nov. 1, 1982 eruption of Sakurajima Volcano

Sakurajima Volcano erupted on November 1, 1982 from 'A-crater'. Iguchi *et al*. (1983) estimated the initial maximum velocity of ejected block as 157 m/s with errors remain less than 10 %, based on the photo analyses. They also observed the distribution of landed blocks which have a diameter larger than 1 m as shown in Fig. 6 (solid circles). The altitude (H) of the bottom of A-crater was

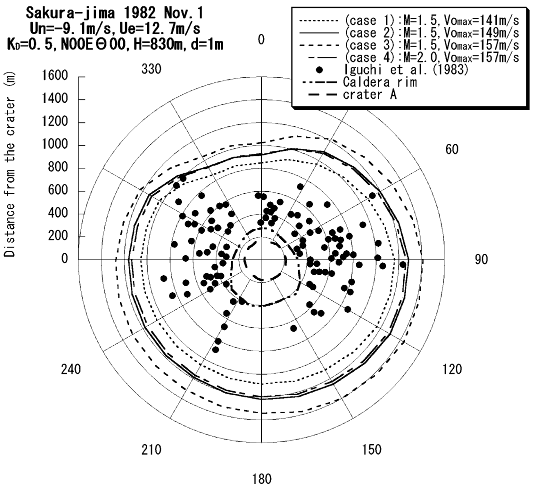


Fig. 6. Comparison between the observed distribution of volcanic blocks in the Nov. 1 1982 eruption of Sakurajima Volcano (solid circles) and calculated ones for various values of parameters.

830 m (a.s.l.). The mean wind velocity from the surface to an altitude of 2 km measured by KLMO was −9.1 m/sec for the N-S component (Un) and 12.7 m/sec for the E-W component (Ue). The northward and eastward components of wind velocity were taken positive.

Applying the trial and error method, we try to re-estimate the initial maximum velocity V_{0max} and the angle dependent parameter M in order to interpret the distribution of the landed blocks that were thrown from A-crater. We use the value of 1 m in diameter of the block for calculations. This value is used commonly for the following analyses of the Jan. 10, 1983 and Oct. 7, 1982 eruptions.

First, we calculate for the following three cases, applying an angle dependent parameter M of 1.5 based on the observations of Iguchi *et al*. (1983) and assuming the different initial maximum velocity to evaluate the range of uncertainty: (case 1) Calculation is made by assuming that an initial maximum velocity V_{0max} is 141 m/s and the explosion principal axis is vertical (N00EΘ00); (case 2) V_{0max} is 149 m/s and the principal axis is vertical (N00EΘ 00); (case 3) V_{0max} is 157 m/s and the principal axis is vertical ((N00EΘ00). Θ denotes supplementary angle of ψ as shown in Fig. 1. The calculated limit of landing distance in the case 2 reproduces well the observed distribution of the landed blocks, while case 1 and case 3 are excluded for candidates of solutions because the calculated limits of landing distance are too narrow or too wide compared to the observed distribution of the landed blocks, respectively (Fig. 6).

Secondly, we calculated the M dependence with the distribution of the landed blocks when we use the observed

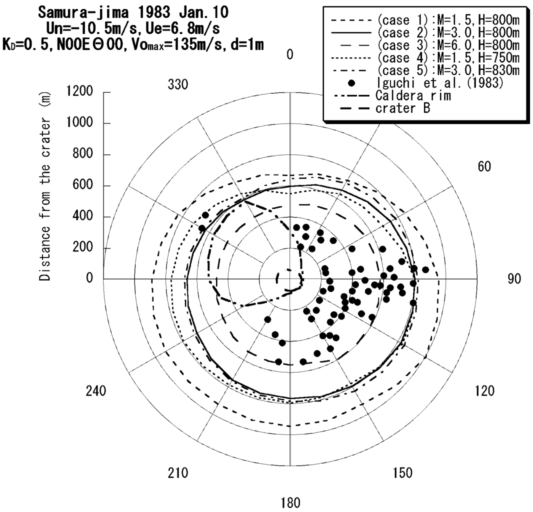


Fig. 7. Comparison between the observed distribution of volcanic blocks in the Jan. 10 1983 eruption of Sakurajima Volcano (solid circles) and calculated ones for various values of M and the altitude H of the bottom of B-crater.

V_{0max} of 157 m/s measured by Iguchi *et al.* (1983). As shown in Fig. 6, $M=2.0$ is suitable for interpreting the observed distribution of the landed blocks (case 4). However, this value of M is in conflict with the observed one as shown in Fig. 2 and described in Section 2-3.

We conclude, therefore, that Sakurajima Volcano predominantly erupted in the vertical direction with an initial velocity of 149 m/s. This value is inside the range of observation error. The angle dependent parameter M is 1.5.

3-2 The Jan.10, 1983 eruption of Sakurajima Volcano

The Sakurajima Volcano erupted on January 10, 1983 from 'B-crater'. Iguchi *et al.* (1983) reported that the altitude H of the bottom of B-crater was not clearly observed but roughly estimated between 700～800 m because the crater had been filled with the dense volcanic gas. They also reported on the distribution of landed blocks which is shown in Fig. 7 (solid circles). In addition, they estimated a maximum initial velocity (V_{0max}) as a reliable value of about 135 m/s through the photo analyses. The observed average wind velocity from the surface to an altitude of 2 km was −10.5 m/sec for the northward component (Un) and 6.8 m/sec for the eastward (Ue) component (after KLMO).

We searched for an appropriate value of M to interpret the distribution of landed blocks using $H=800$ m and $V_{0max}=135$ m/s. As the crater is narrow and is surrounded by steep and vertical wall, we assumed that the explosion principal axis is vertical (N00EΘ00). Among trial and error calculations, Fig. 7 shows the comparison of the typical distribution of landed blocks and the calculated limits of landing distance for three cases: (case 1) $M=1.5$; (case 2) $M=3.0$; (case 3) $M=6.0$. Case 3 cannot reproduce the distribution of landed blocks because the calculated limit of landing distance is too narrow to satisfy the observed distribution. The observed outer limit of landed blocks is distributed in between the calculated ones for case 1 and case 2.

Next, we investigated the effect of the altitude H to the distribution of landed blocks because the bottom of the B-crater was not clearly observed as mentioned above. Then, we assume the following two cases: (case 4) $M=1.5$; $H=750$ m; (case 5) $M=3.0$; $H=830$ m which is the same altitude as that of A-crater. The calculated results shown in Fig. 7 reveal that case 2 and case 5 are not distinguishable and we prefer case 4 as a suitable model to reproduce the distribution of landed blocks.

In conclusion, the Jan. 10, 1983 eruption of Sakurajima probably ejected almost vertically with the maximum initial velocity of 135 m/s at an altitude of about 750 m. The present analyses also suggest that the angle dependent parameter M is considerably smaller than the re-examined value of 6.0 described in Section 2.3 and estimated as 1.5. This discrepancy may be derived from a small number of data.

3-3 The Oct. 7, 1982 Eruption of Sakurajima Volcano

The Sakurajima Volcano erupted at A-crater on October 7, 1982. Unfortunately, the initial velocity of eruptions was not observed for this activity and only the distribution of landed blocks was observed by Iguchi and Kamo (1984) as shown in Fig. 8 (solid circles). They proposed a model to reproduce the distribution of landed blocks assuming that the explosion principal axis has a tilt of 15° from the vertical axis to the direction of N82°E direction (N82EΘ 15) and the maximum initial velocity was 120 m/s.

In addition, we search for other possible combinations of parameters to reproduce the distribution of blocks under the conditions of altitude $H=830$ m and the observed wind velocity $Un=-5.0$ m/s and $Ue=-4.9$ m/s (after KLMO). For example, assuming a combination of the initial velocity V_{0max}, direction of the explosion principal axis and the angle dependent parameter M, we calculate the distribution of blocks for the following cases such as (case 1: $V_{0max}=126$ m/s, the explosion principal axis is N82EΘ 15, $M=1.5$), (case 2: 140 m/s, N82EΘ13, 3.0), and (case 3: 164 m/s, N82EΘ10, 6.0). As a result, any case with various combinations of parameters almost satisfies the observed distribution of blocks as shown in Fig. 8. Therefore, it may be difficult to estimate the values of the parameters with only the data of the distribution of the landed blocks.

3-4 Expected information for estimating constrained value of parameters

We described in the present paper that the motion of the

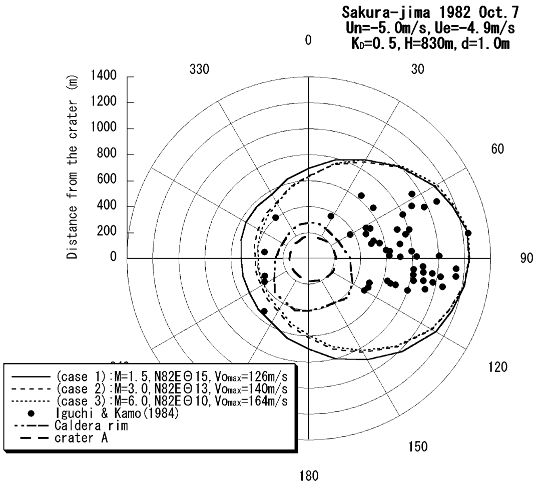


Fig. 8. Comparison between the observed distribution of volcanic blocks at the Oct. 7, 1982 eruption of Sakura-jima Volcano (solid circles) and calculated ones for various values of parameters.

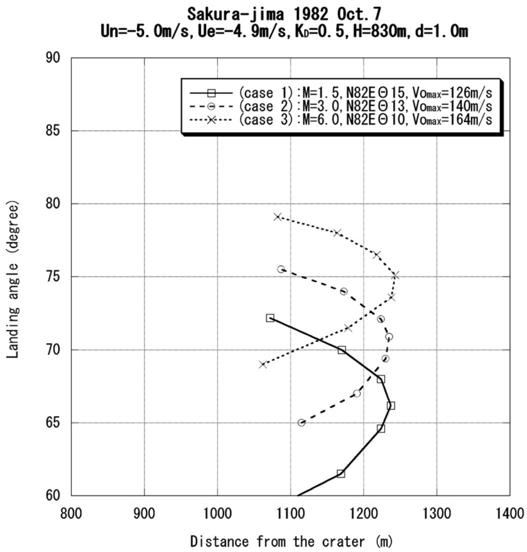


Fig. 9. Calculated relation between landing distances and landing angles for the parameters used in the cases 1～3 in Section 3-3.

ejected blocks is controlled by many physical parameters such as the initial velocity and direction of explosion, the angle dependent parameter and so on. However, it is often difficult to make direct observation of these parameters. Direction and initial velocity of explosion were relatively well constrained in the case of the November 1, 1982 eruption as described in Section 3-1 because the initial maximum velocity V_{0max} and the angle dependent parameter M were measured. In comparison, only rough estimations of these parameters were made in the case of the Jan. 10, 1983 eruption because the reliability of the angle dependent parameter M was not necessarily assured. As various combinations of parameters satisfy the distribution of the landed blocks, appropriate values of the direction and initial velocity of explosion were not determined in the case of the October 7, 1982 eruption.

To overcome the above-mentioned difficulties for estimating the parameters, we propose additional approaches to be taken during the next eruption: (1) If we could measure landing angle of the flying block, it would help us to constrain the combination of parameters. Fig. 9 shows examples of calculated relations between the landing angles and the landing distances from A-crater in the eastward direction. The same values of parameters as the cases 1～3 in Section 3-3 are applied in this calculation. The calculated three cases are readily distinguishable. In an actual field, Minakami (1942) obtained the landing angles of three bombs at the 1938 eruption of Asama-yama Volcano by measuring traces of stuck bombs. This case proved several degrees were distinguishable. (2) Systematic investigation of detailed size distribution of the landed blocks is needed to prove the diameter dependence of the initial velocity.

4. Conclusions

We presented an improved equation of motion considering the air resistance, wind velocity, and direction dependence of the initial velocity. It is noteworthy that we treated the air resistance as a vector that coincides with the opposite direction of flight.

Using this equation, we numerically calculated synthetic ballistic curves for various combinations of the above-mentioned effects to reproduce the motion of the blocks at the 1982 and 1983 eruptions of Sakurajima Volcano. Referring to the observed distribution of landed blocks and photo data of the explosion, we concluded that the explosion on Nov. 1, 1982 was predominantly in the vertical direction with an initial velocity of around 149 m/s and the angle dependent parameter M is 1.5. In case of the Jan. 10, 1983 eruption, the observed distribution of landed blocks and the photo data of the explosion were also provided for analysis. The principal direction of explosion was probably vertical with a maximum initial velocity of 135 m/s at an altitude of about 750 m. The angle dependent parameter M was estimated as 1. 5 which is considerably smaller than the previously observed value of 6.0. This discrepancy may be derived from small numbers of observed data. Unfortunately, it was difficult to estimate the values of the parameters in case of the Oct. 7, 1982 eruption because only the distribution of landed blocks was observed.

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放出岩塊の弾道方程式の改良と 1982・1983 年桜島噴火への適用

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本論文では，火山爆発による放出岩塊の初速度の方位とサイズの依存性を考慮し，空気抵抗をベクトルとして扱う新たな弾道方程式を提案した．本方程式を桜島南岳爆発の岩塊落下地点分布に適用したところ，1982 年 11 月 1 日，および 1983 年 1 月 10 日の噴火の最大初速度としてそれぞれ，149 m/s と 135 m/s が得られ，いずれもほぼ鉛直方向に爆発したことが推定された．一方，爆発の主軸が傾く 1982 年 10 月 7 日の噴火では，岩塊分布を説明可能なパラメータの範囲が広いことが分かった．